

# A Tinkertoy<sup>(Rg. Tr.)</sup> Model that Walks

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Submitted to *Physical Review Letters*, June 14, 1997

(July 12, 1997)

We describe a simple two-legged Tinkertoy<sup>(Rg. Tr.)</sup> device that walks, apparently stably, and somewhat human-like down gentle slopes. It is powered only by gravity and has no control. This 3D walking device seems to be the first McGeer-like passive-dynamic walker that is statically unstable in every standing position, yet dynamically stable. It highlights the possible role of uncontrolled dynamics in side-to-side balance. It joins a small set of mechanical devices of any kind that are dynamically stable at or near a statically unstable configuration but do not depend on gyroscopic effects.

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## I. INTRODUCTION

Inspired by the calculations on swinging legs by Mochon and McMahon [1,2] and by simple cereal-box walking toys, McGeer [3-5] successfully sought and found two-dimensional, straight-legged and kneed walking model designs that displayed graceful, stable, human-like walking on a range of shallow slopes with no actuation (besides gravity) and no control. McGeer termed the motions of these machines *passive-dynamic* walking. Unlike the cereal-box toys, McGeer's robots cannot stand upright and still; their balance depends intrinsically on their dynamics. All of McGeer's successful designs, as well as those of his imitators thus far, Garcia *et al.* [6], Coleman [7], and Goswami *et al.* [8], have been more-or-less constrained against falling over sideways so that their dynamic balance was fore-aft only.

All of the passive-dynamic walking toys that we know about are statically stable so that their dynamic stability is not so great a surprise.

While locomotion occurs almost exclusively in the sagittal (fore-aft) plane, the stability of out-of-plane motions is an important issue. McGeer began studies of three-dimensional passive walking mechanisms finding only unstable periodic motions. Fowlie and Kuo [9] numerically simulated a passive-dynamic 3D model of walking simpler than McGeer's (the model could bank and pitch but not steer), but did not find stable passive motions so focussed their attention on stabilizing motion using simple feedback control laws that govern foot and torso placement.

Other than our Tinkertoy<sup>(Rg. Tr.)</sup> device, we know of only two uncontrolled three-dimensional devices that can have asymptotically stable steady motions at or near a potential energy maximum, without depending on gyroscopic effects. One is a bicycle with massless wheels (say skates) and a special mass distribution [10]. The other device is a stiff rider attached to a skate-board [11]. These two devices differ from the walking toy we present here in that they are constrained against fore-aft tipping (ours is free to fall), they conserve energy (ours loses energy at impact and uses up gravitational potential energy), and they are non-holonomically constrained (ours is piecewise holonomic). Certain gliding aircraft might be considered as a third example, but the definition of a potential energy maximum is less clear for them.

Our investigations into passive 3D walking devices began with a rigid rimless spoked wheel, or regular polygon, free to 'roll' downhill. The 3D rimless wheel shares some qualitative features with passive-dynamic walking machines; it is a passive three-dimensional system with intermittent impacts and periodic motions but without swinging legs. At any fixed, large enough slope, we found that the rimless spoked wheel has asymptotically stable limit cycle or steady 'rolling' motions [12]. This differs from the classical case of a uniform rolling disk which, because of its symmetry, has only neutral stability. Both the disk and the rimless wheel depend on gyroscopic effects for stability. But, we do not yet know how the stronger asymptotic stability of the 3D rimless wheel depends on its intermittent contact.

## II. DESCRIPTION OF OUR DEVICE

Our physical model is constructed from standard Tinkertoy<sup>(Rg. Tr.)</sup> pieces and metal nuts (for weight). It has two straight legs, separated by two hinges at the hip joints, and rounded feet (see Fig. 1). The legs have low mass centers located laterally at a distance comparable to the leg length. The device takes tiny steps and moves by rocking side-to-side coupled with swinging of the legs.

Fig. 2 shows a 3D model which captures what we believe to be the essential geometric and mass-distribution features of the Tinkertoy<sup>(Rg. Tr.)</sup> model. That is, the behavior should be well described by a model based

on hinged-together rigid bodies making collisional and rolling contact with the ground. The model has two symmetric legs with general spatial mass distribution (which includes the foot mass), ellipsoidal feet, and frictionless hinges at each hip joint. We do not yet know which of these features are essential.

When a foot hits the ground, the velocity of the contact point jumps instantaneously to zero. The foot rolls without slip, until the swinging foot reaches heel-strike. Except at the instant of foot collision, only one foot is in contact with the ground at any time. When the swinging foot collides with the ground, the trailing leg is assumed to leave the ground non-impulsively at the same instant.

Following McGeer, in order to study the stability of such systems, we represent an entire gait cycle (two leg swings and two foot collisions) by a Poincaré map which we evaluate by numerical integration of the differential equations and jump conditions derived from linear and angular momentum balance. Fixed points of the return map correspond to gait cycles (not necessarily stable). We find fixed points by a numerical search or path following proceeding from fixed points found with other parameter values. We determine the stability of periodic motions by numerically calculating the eigenvalues of the linearization of the return map at the fixed points. If the magnitudes of all of the eigenvalues are smaller than one, then the fixed point is asymptotically stable.

Currently, we have been studying a simplification with  $r_1 = r_2 = 0$  and  $w = 0$ ; i.e., a point-foot model with two rigid body legs connected by a hinge joint at the hip. In this case, when a foot is on the ground, the contact acts like a ball-and-socket joint. To date, like McGeer [5] and Fowble and Kuo [9], we have found only unstable periodic motions, though much less unstable than theirs. A nearly stable case from our numerical studies has maximum eigenvalue of about 1.15, one eigenvalue exactly one, and all others less than one. The eigenvector associated with the maximum eigenvalue points in the direction of falling over sideways, as expected (since fore-aft balance has already been achieved by two-dimensional walkers). The most stable mass distributions we have found thus far do not have very human-like parameters; each leg has a center of mass close to the floor, although still above the foot centers, and laterally displaced at about 90% of leg length.

In this case, the walker's legs have a mass distribution corresponding roughly to laterally extended balance bars, like what might be used for walking on a tight-rope. Clearly, in the limit as lateral mass distribution and lateral offset of the center of mass get very large, the modulus of the maximum eigenvalue asymptotically approaches one, or neutral lateral stability, from above.

## The Tinkertoy<sup>(Rg. Tr.)</sup> Device Construction

Based on the numerical results, we have constructed, initially as a non-working demo and not for walking experiments, a device similar to the one shown in Fig. 1. On somewhat of a caprice, we tested the first walking device on a ramp and, surprisingly, it nearly took a few serendipitous, if not very steady or stable, steps. After some tinkering, we arrived at the device shown. The structural parts for our 3D walking device come from a Playskool<sup>(Rg. Tr.)</sup> Tinkertoy<sup>(Rg. Tr.)</sup> Construction System: Colossal Constructions<sup>TM</sup> (1991 set) and the balance masses are assembled from various steel nuts. We glue together rigidly a yellow spool, a green rod, and a green hinge, to act as a leg and foot, as shown in Fig. 1. Then we slide the legs onto a red rod which acts as an axle; the hinges are separated by a friction-fit orange washer and constrained from sliding off of the axle by two other friction-fit washers. We glued a yellow spool rigidly to the end of a red rod and insert the other end into the side of a yellow foot (see Fig. 1) but only with a friction fit to allow for rotational adjustment. We assemble each balance mass from three nuts stacked vertically and held rigidly together between two washers by a nut and bolt. Then each mass assembly is located on the yellow spools at the end of the balance rods and held in place with black vinyl electrical tape. One important construction note: because a yellow spool has holes located radially around its circumference to accept rods, a small flat section is left as a result of the fabrication process; thus, to ensure that the walker is statically unstable (cannot stand on the flat sections or in any other way), a small strip of thin brass spring material is fastened over the flat section contacting the floor so as to restore its curvature there (see Fig. 1). Finally, the walking ramp has about a 5 degree slope.

## III. CONCLUSIONS AND FUTURE WORK

We have built and demonstrated an apparently stable physical 3D passive-dynamic walking model, loosely based on the results of numerical simulations. It is yet to be determined what aspects of a mechanical model are needed to achieve such walking.

Although our new machine does not have a very human-like mass distribution, it does highlight the possibility that uncontrolled dynamics may not just contribute to fore-aft walking balance, as indicated by previous McGeer models, but also to side-to-side balance.

The mechanism is also distinctive in that it joins a small collection of devices which dynamically balance without any rapidly spinning parts; i. e., it works without gyroscopic effects.

Conservative *and* nonholonomic systems (non-Hamiltonian) with appropriate mass distributions can have asymptotically stable, steady motions. An open and possibly unanswerable question is whether the stability of this intermittently dissipative system can be explained, in part, by the fact that its piecewise holonomic constraints act somewhat like non-holonomic constraints; i. e., the accessible configuration space is bigger in dimension than that of the accessible velocity space.

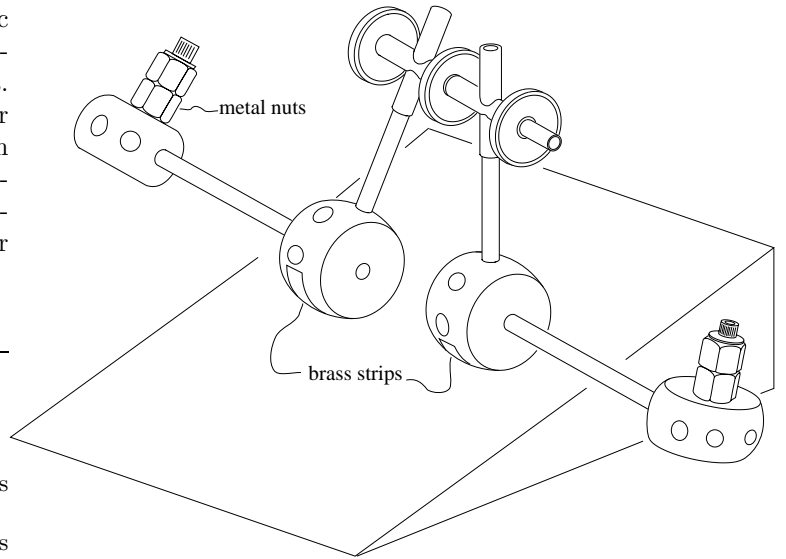


FIG. 1. A pictorial sketch of our 3D Tinkertoy<sup>(Rg. Tdmk.)</sup> walking model. The center of mass of the device is above the centers of the wheel-like feet and behind the leg axes. The metal nuts for weight and the brass strips to round the foot bottoms are fastened with black electrical tape (not shown).

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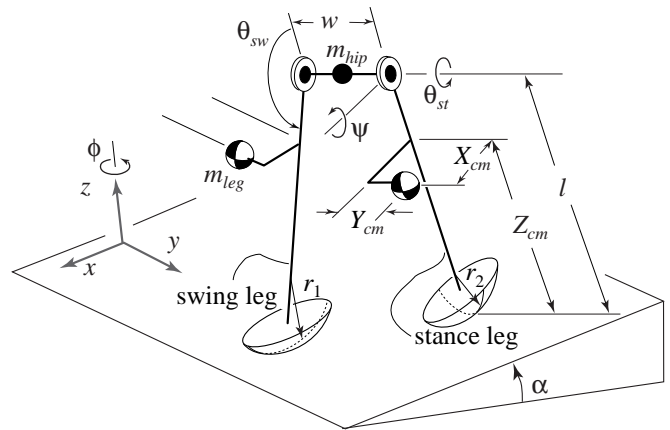


FIG. 2. A rigid body model of the tinkertoy walker: parameters and configuration variables. The legs have length  $\ell$ , mass  $m_{leg}$ , moment of inertia matrix  $\mathbf{I}^m$ , and center of mass location  $(X_{cm}, Y_{cm}, Z_{cm})$ . The hip joints are separated by a distance  $w$ . The principal radii of curvature of both feet are  $r_1$  and  $r_2$ . The slope angle is  $\alpha$  and the acceleration due to gravity is  $g$ . The orientation of the stance leg with respect to an inertial frame is given in terms of 3-1-2 Euler angles; the heading angle  $\phi$  about the original  $z$ -axis, the bank angle  $\psi$  about the new  $x$ -axis, and the stance-leg pitch angle  $\theta_{st}$  about the newest  $y$ -axis. The swing leg pitch angle  $\theta_{sw}$  is measured with respect to the stance leg about the hip axle. The swing and stance legs have the same heading and bank angles.